



Division of Strength of Materials and Structures

Faculty of Power and Aeronautical Engineering



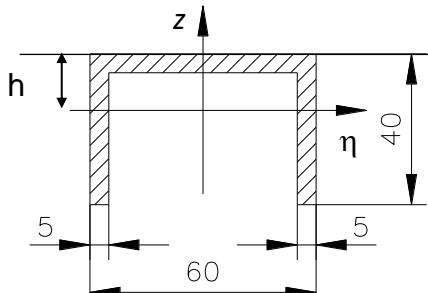
# Finite element method (FEM1)

Lecture 13A. Examples of FE modeling

05.2025

# Example 1. Cantilever beam

$P=1000 \text{ N}$



$$E = 2 \cdot 10^5 \text{ MPa}$$

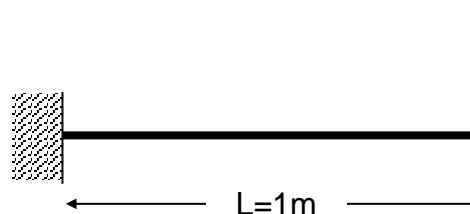
$$\nu = 0.3$$

$$\rho = 8 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$$

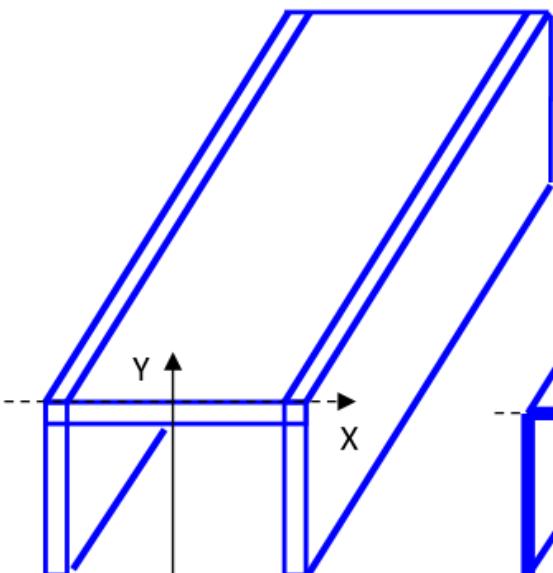
$$J = J_\eta = 1.0097 \cdot 10^{-7} \text{ m}^4$$

$$A = 0.6 \cdot 10^{-3} \text{ m}^2$$

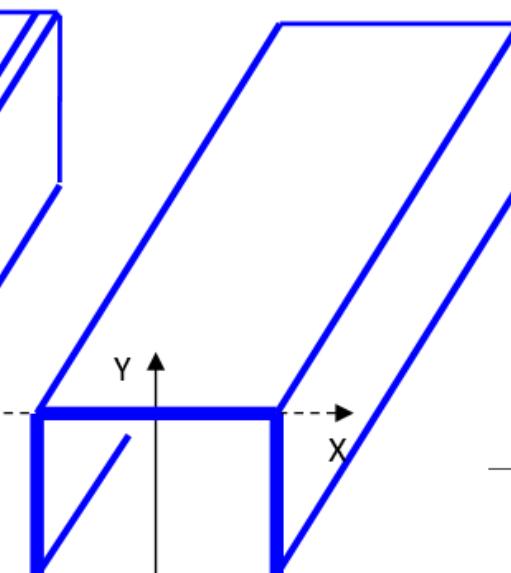
$$h = 13.27 \text{ mm}$$



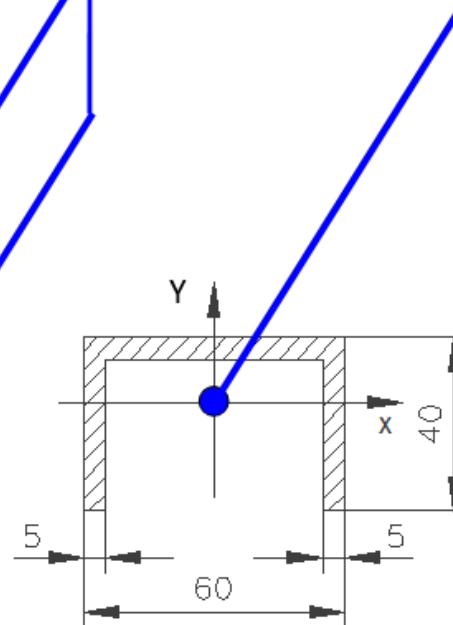
Solid 185



Shell 181



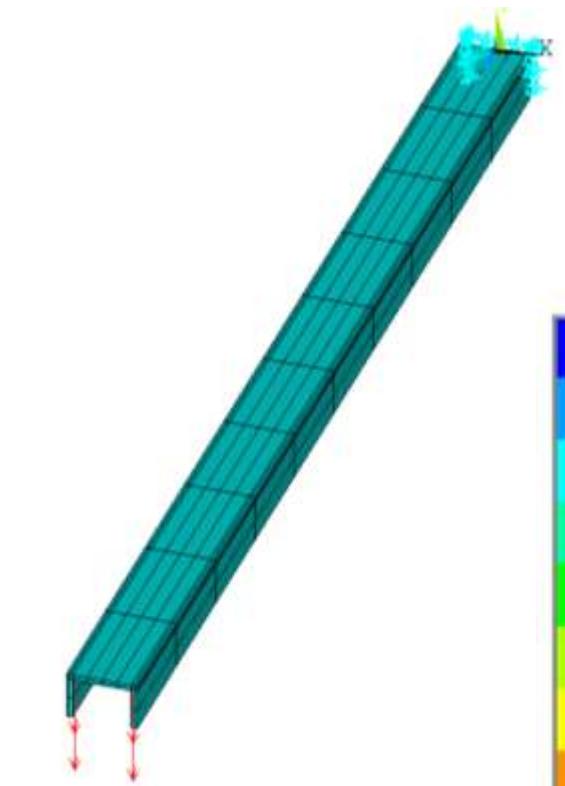
Beam 188



Extruding surfaces produces solids filled with solid elements (3D modeling)

Extruding lines gives areas that we will cover with shell elements of a given thickness

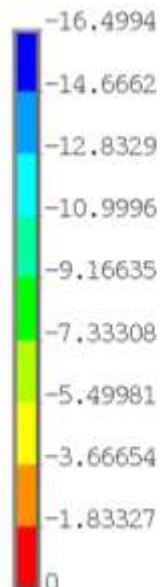
Extracting a point gives a line to which we assign a section defining the features of the cross section



# Solid 185

Nodes . .  
Elements.

429  
240



STEP=1  
SUB =1  
TIME=1  
UY  
RSYS=0

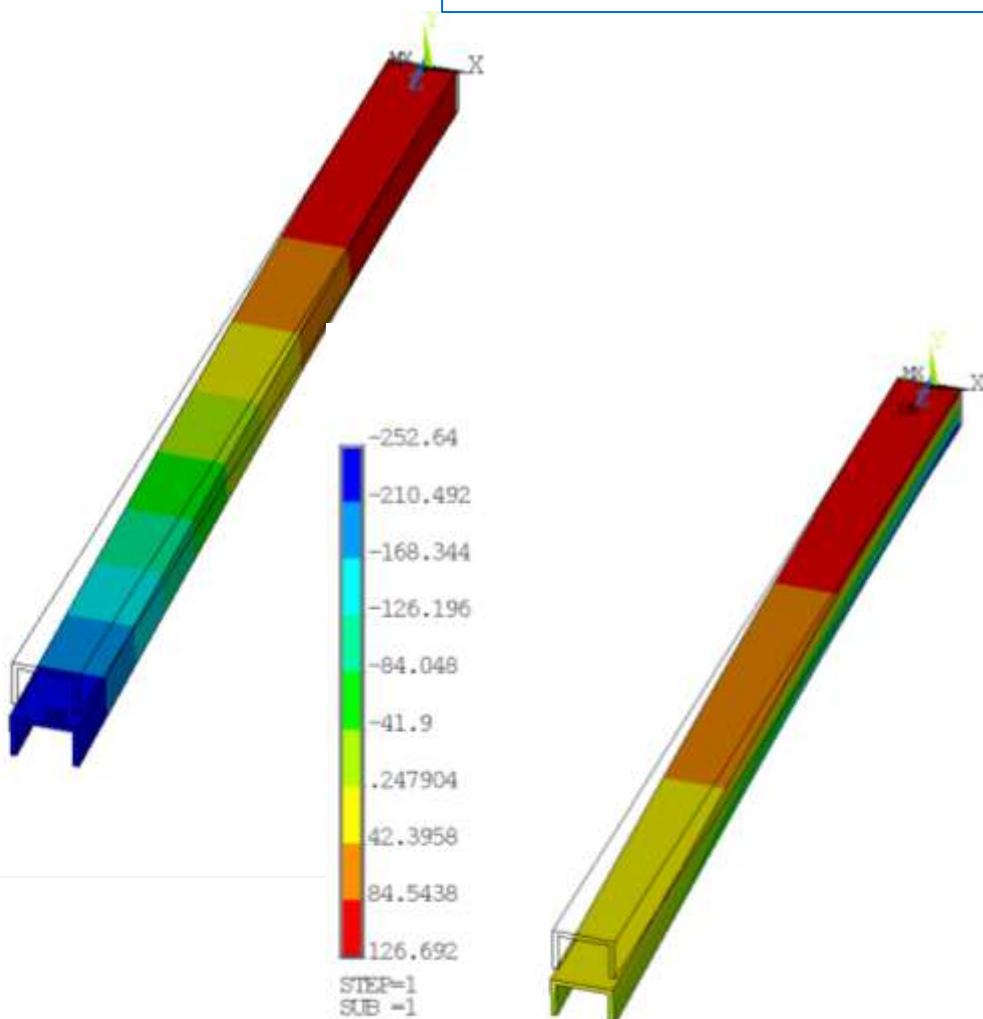
(AVG)

BELKA\_3D

## Analytical results:

$$f = PL^3/(3EJ) = 16,5\text{mm}$$

$$\sigma = +132 / -265 \text{ MPa}$$



-252.64  
-210.492  
-168.344  
-126.196  
-84.048  
-41.9  
.247904  
42.3958  
84.5438  
126.692

STEP=1  
SUB =1  
TIME=1  
SZ  
RSYS=0

(AVG)

BELKA\_3D



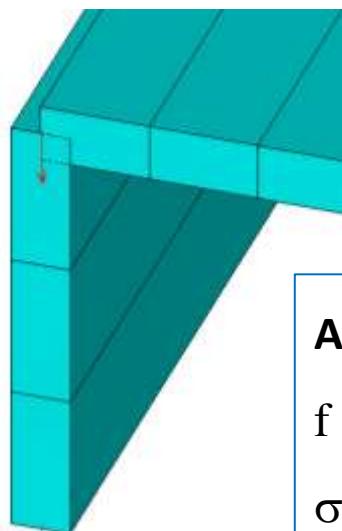
**Shell 181**

Nodes . .	143
Elements.	120



STEP=1  
SUB =1  
TIME=1  
UY (AVG)  
RSYS=0

BELKA\_Shell181



**Analytical results:**

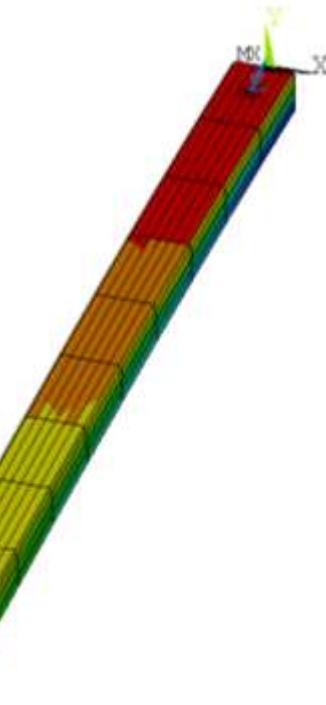
$$f = PL^3/(3EJ) = 16,5 \text{ mm}$$

$$\sigma = +132 / -265 \text{ MPa}$$



STEP=1  
SUB =1  
TIME=1  
SZ (AVG)  
RSYS=0

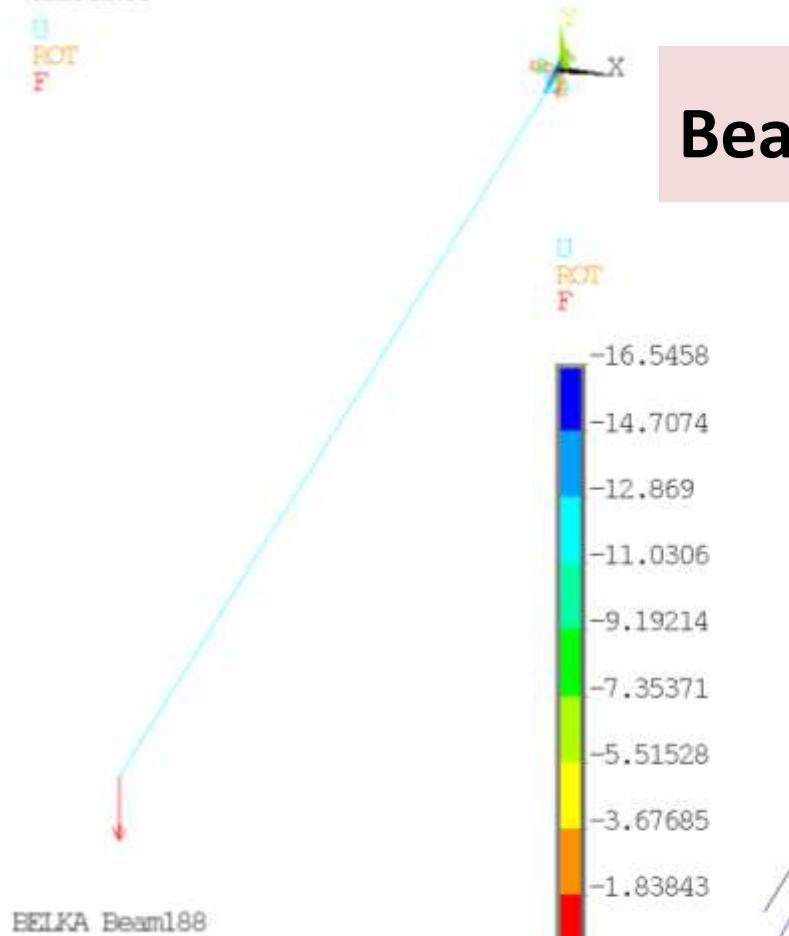
BELKA\_Shell181



U  
ROT  
F

## Beam 188

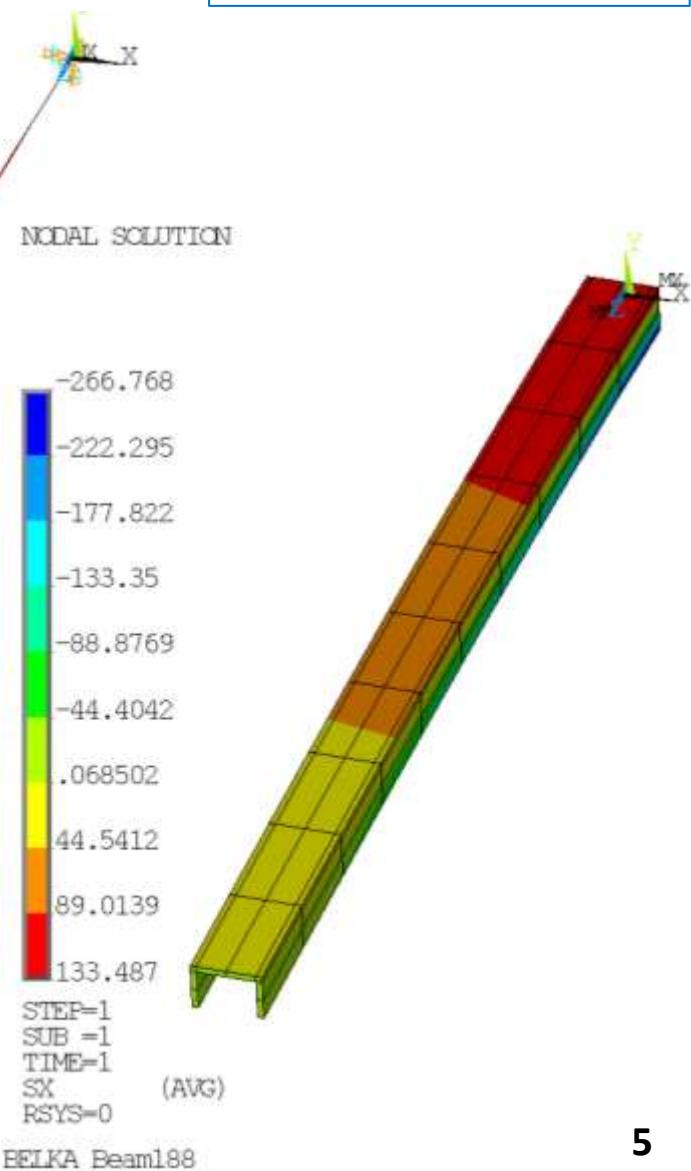
Nodes . . . 31  
Elements . . 10

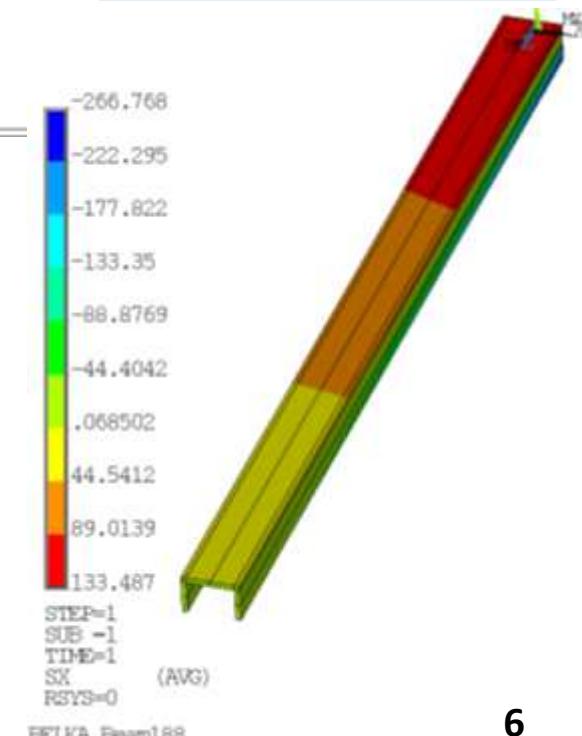
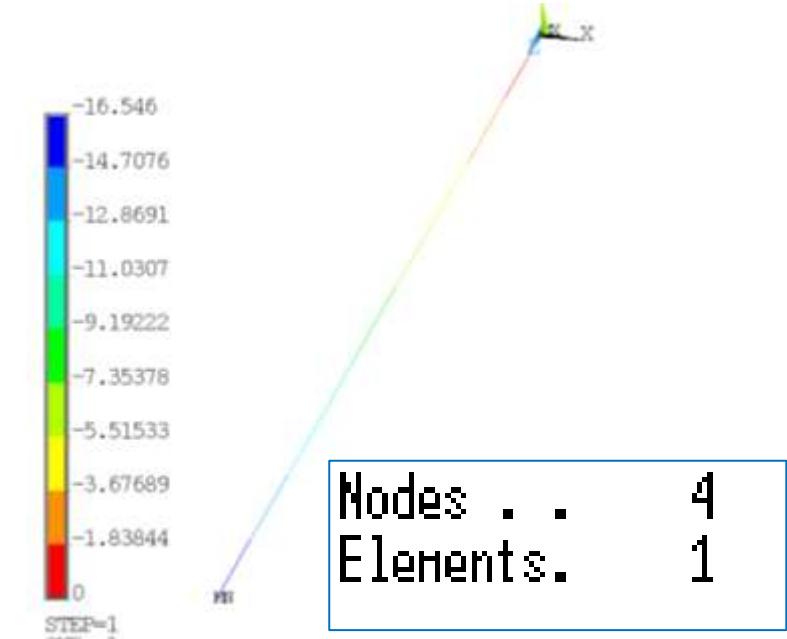
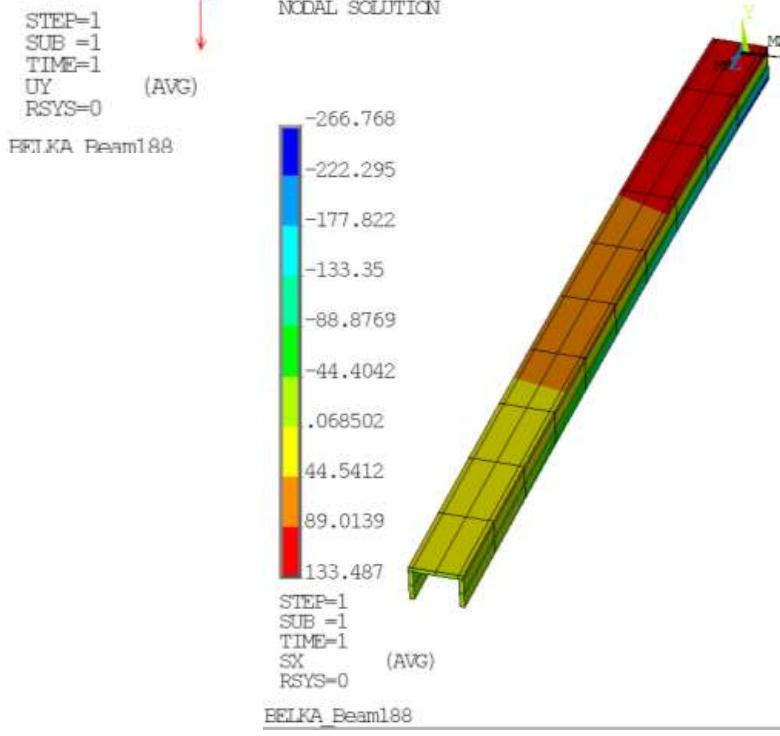
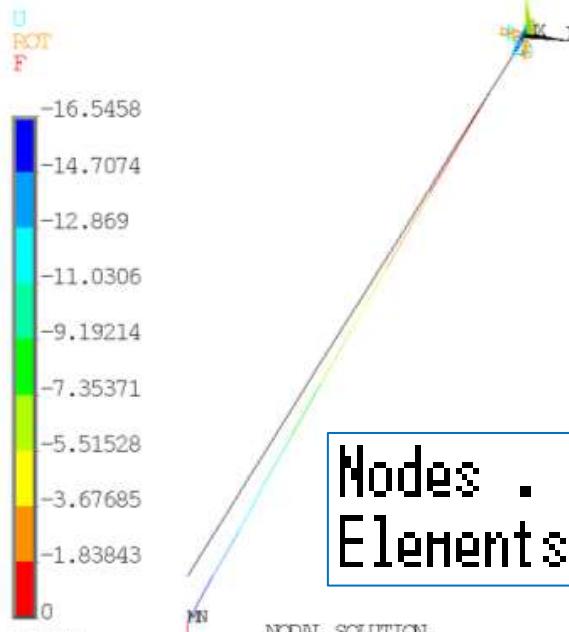


### Analytical results:

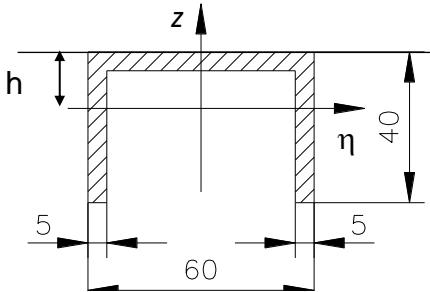
$$f = PL^3/(3EJ) = 16,5\text{mm}$$

$$\sigma = +132 / -265 \text{ MPa}$$

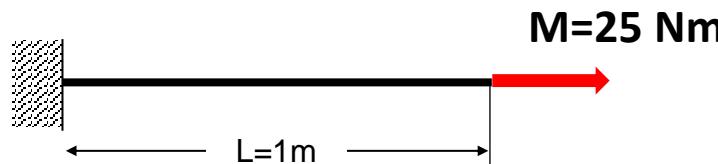




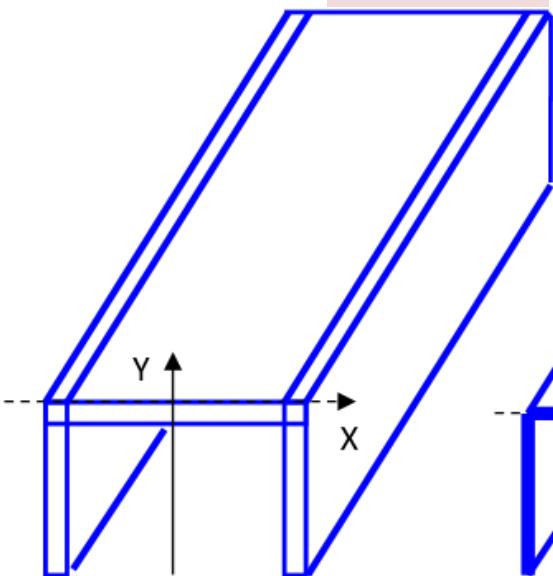
## Example 2. Twisted rod



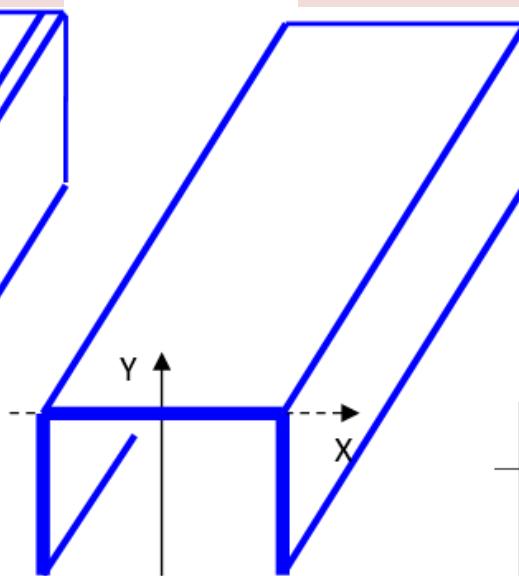
$E = 2 \cdot 10^5 \text{ MPa}$   
 $\nu = 0.3$   
 $\rho = 8 \cdot 10^3 \text{ kg/m}^3$   
 $J = J_\eta = 1.0097 \cdot 10^{-7} \text{ m}^4$   
 $A = 0.6 \cdot 10^{-3} \text{ m}^2$   
 $h = 12.27 \text{ mm}$



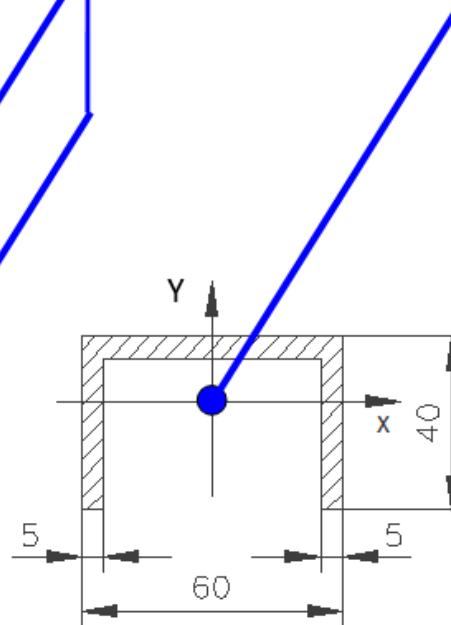
Solid 185

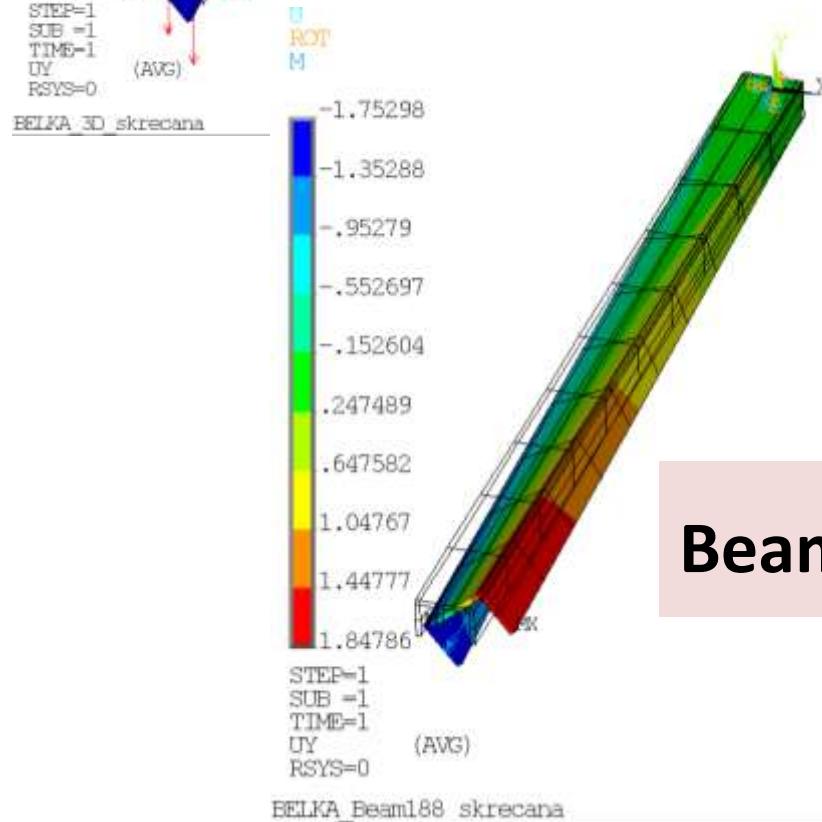
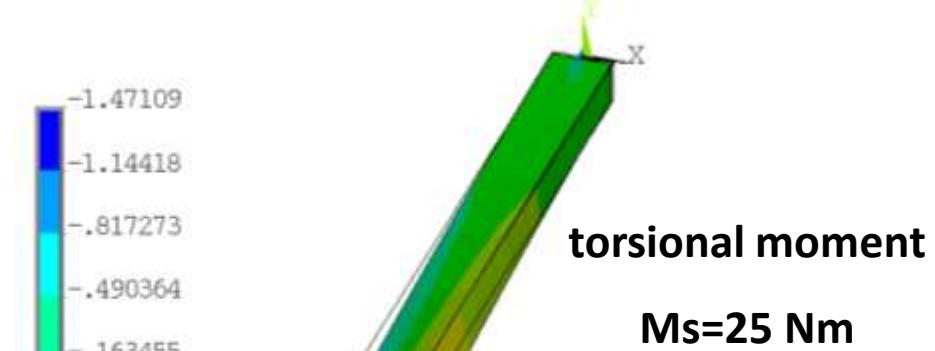
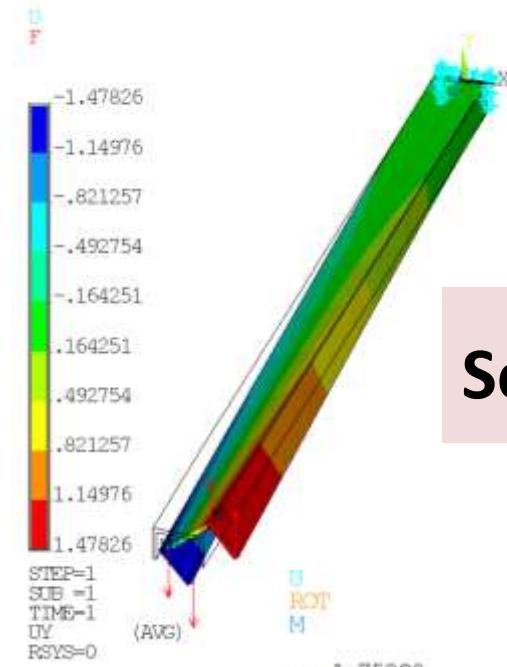


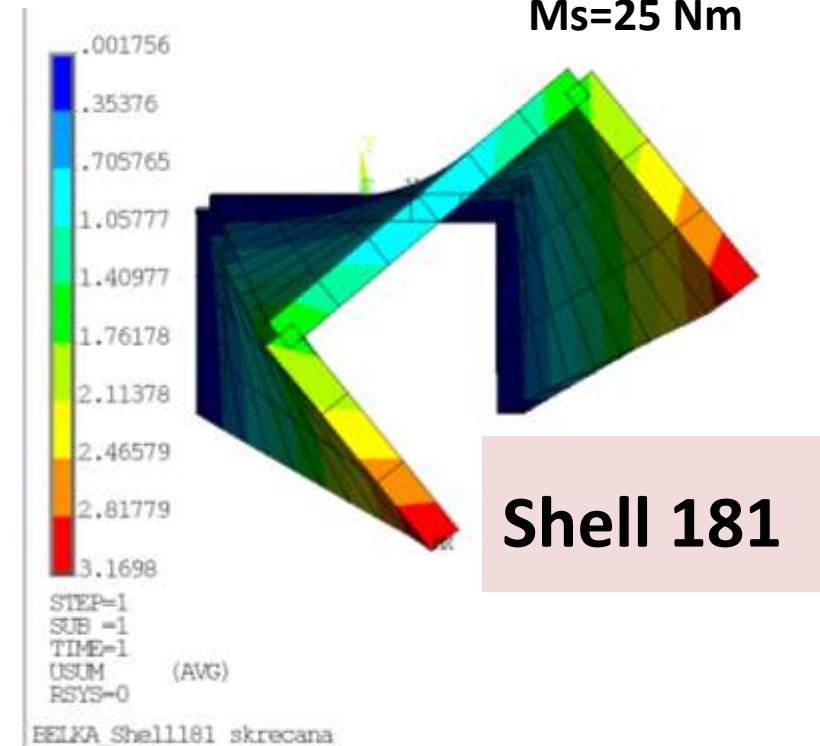
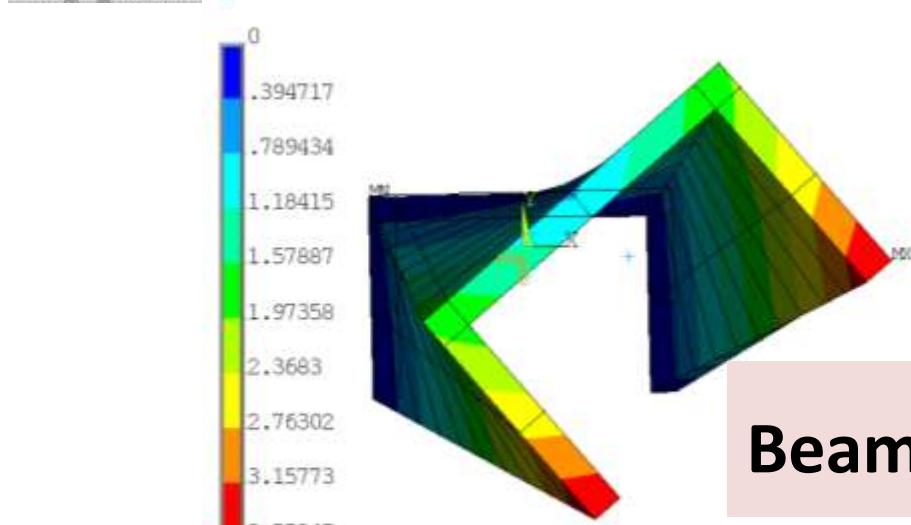
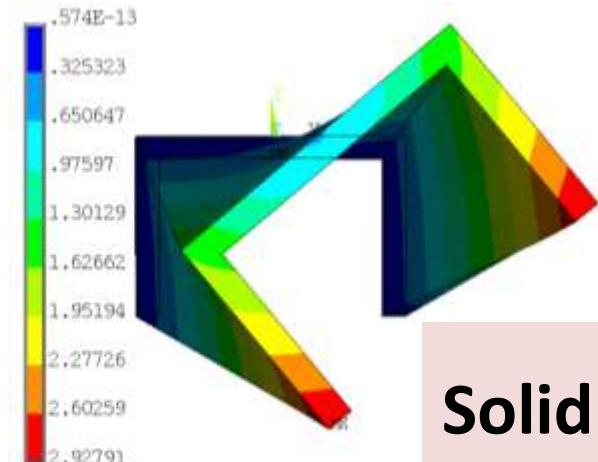
Shell 181



Beam 188

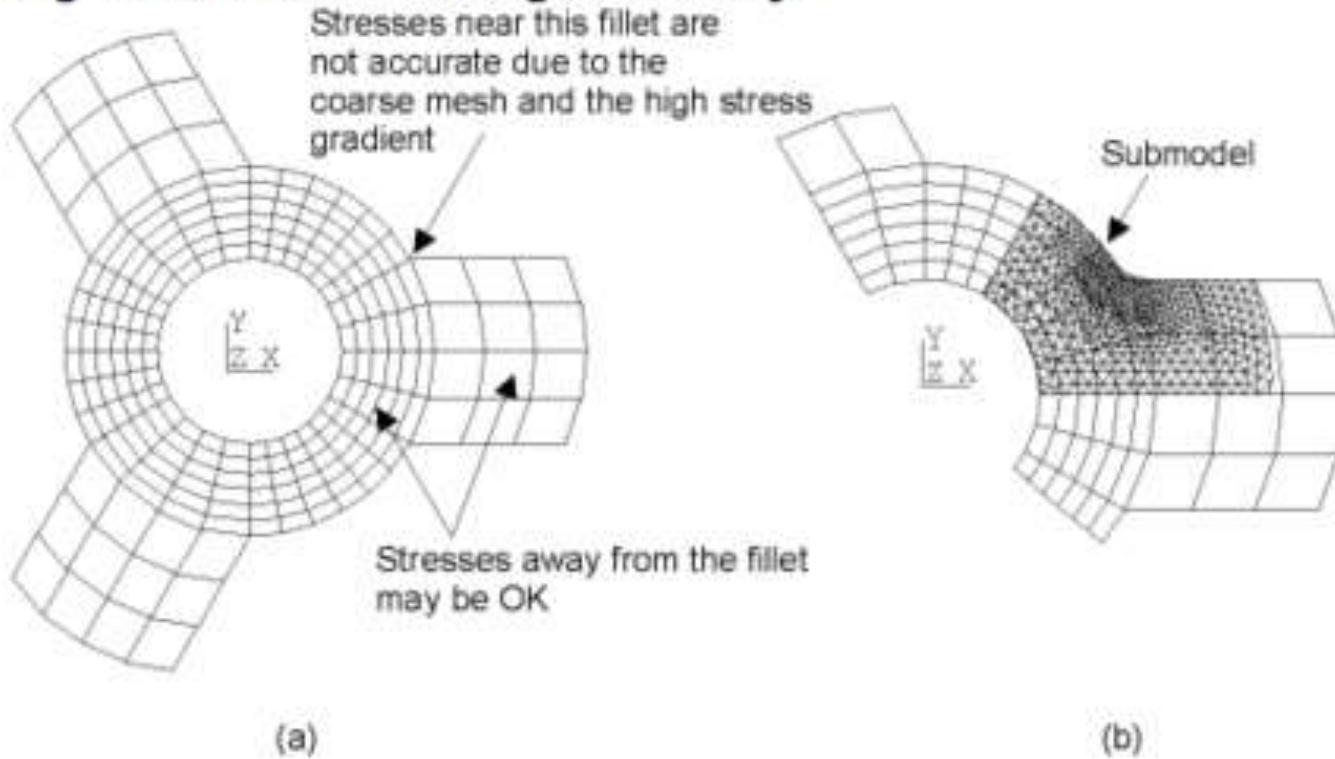






# Submodeling

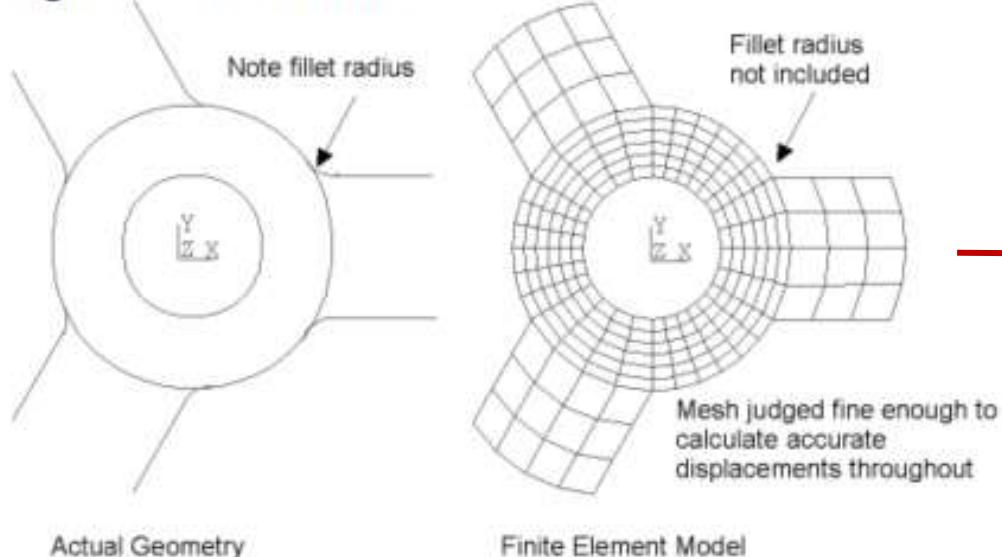
**Figure 9.1: Submodeling of a Pulley**



Submodeling of a pulley hub and spokes: (a) coarsely meshed model, and  
(b) finely meshed submodel (shown superimposed over coarse model)

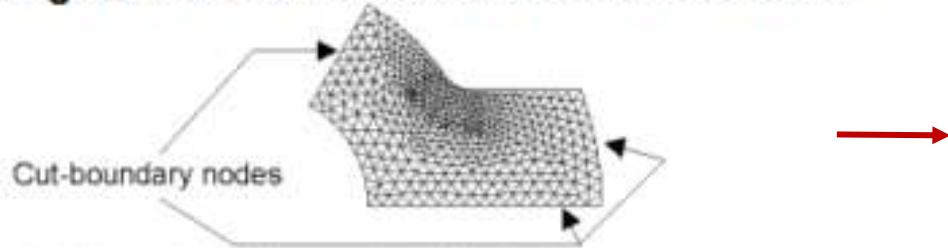
# Understanding Submodeling

**Figure 9.2: Coarse Model**

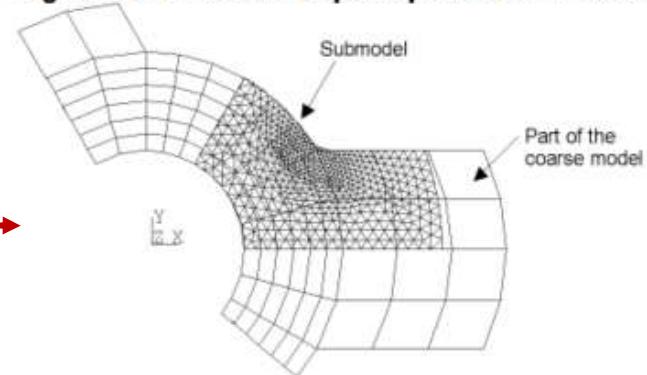


Initial, coarse model may not need to include many details

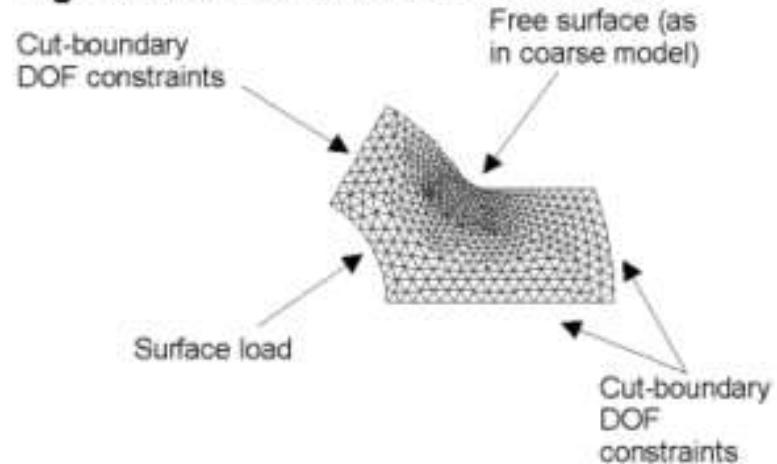
**Figure 9.4: Cut Boundaries on the Submodel**



**Figure 9.3: Submodel Superimposed Over Coarse Model**

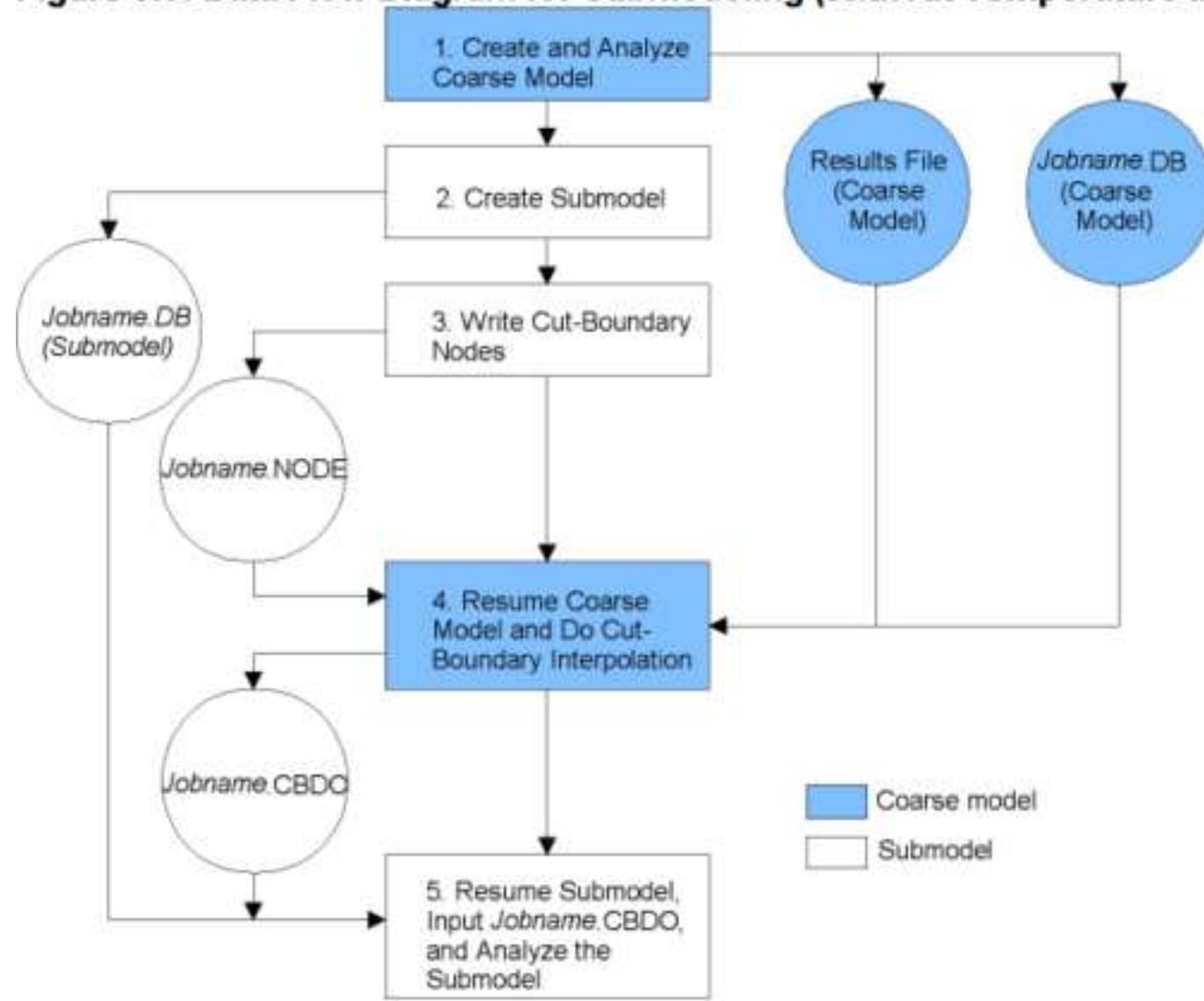


**Figure 9.5: Loads on the Submodel**



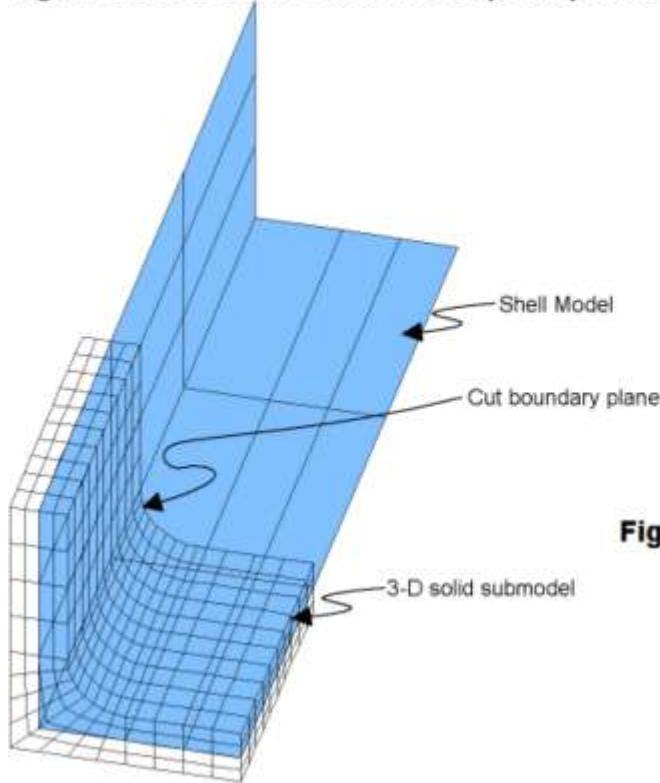
## Understanding Submodeling

**Figure 9.6: Data Flow Diagram for Submodeling (Without Temperature Interpolation)**



# Shell-to-Solid Submodels

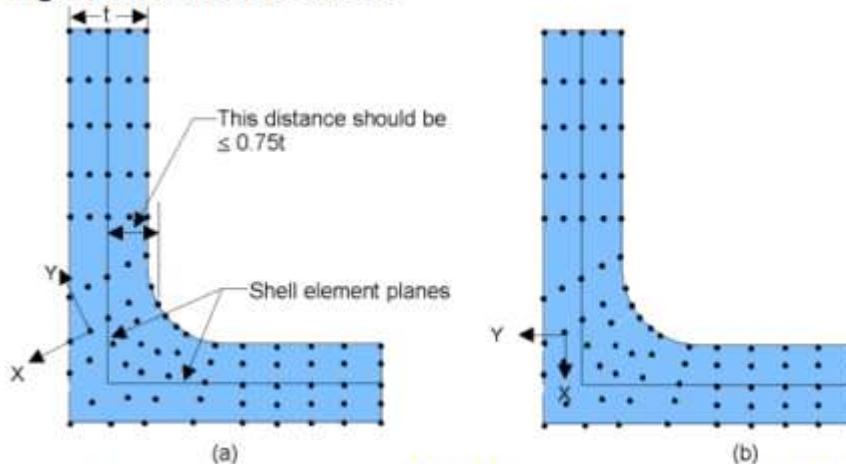
Figure 9.9: 3-D Solid Submodel Superimposed on Coarse Shell Model



In a structural analysis, only translational displacements are calculated for the cut boundary nodes, but their values are based on both the translations and rotations of the projected point.

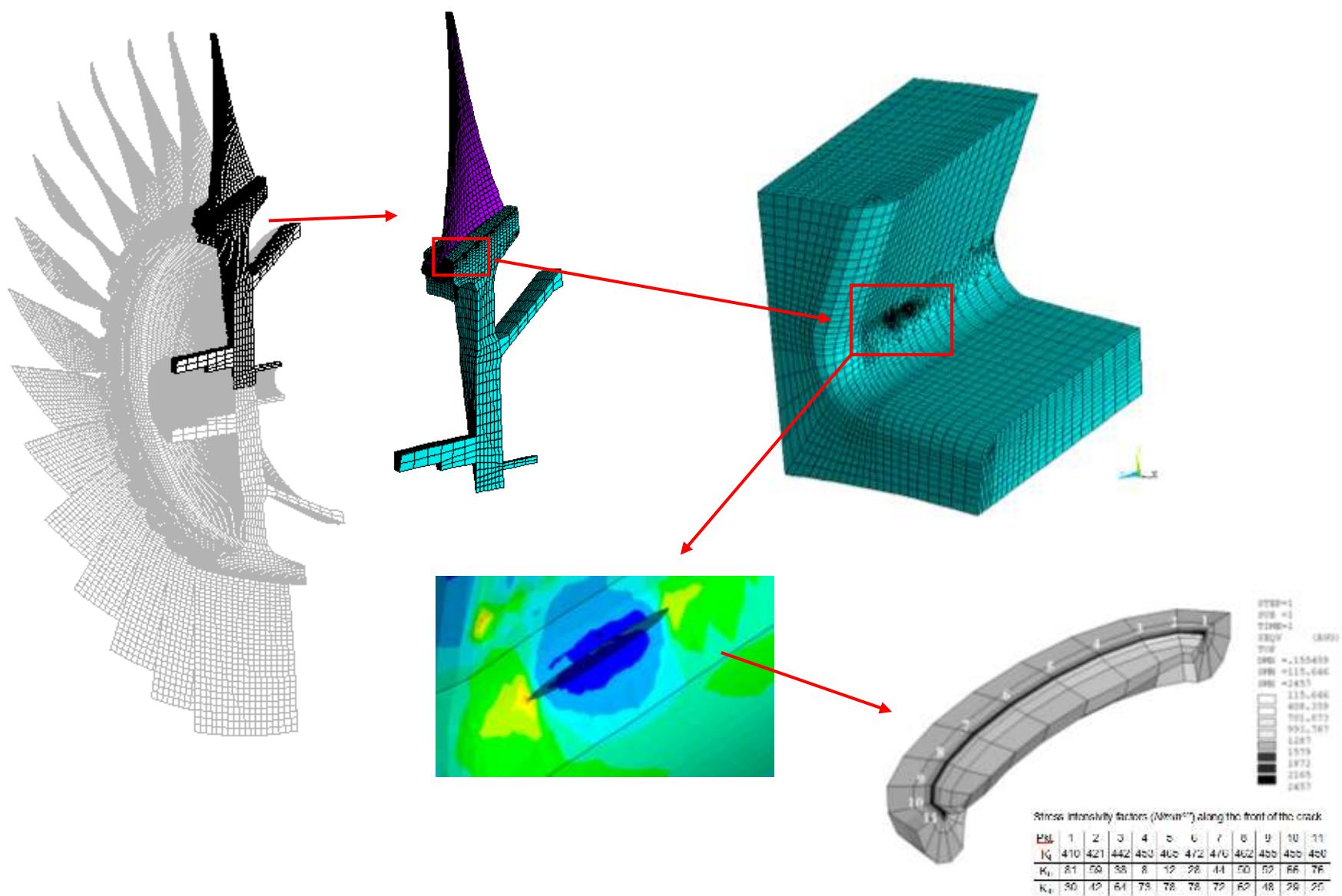
Also, the node is rotated such that the nodal UY direction is always perpendicular to the shell plane

Figure 9.10: Node Rotations

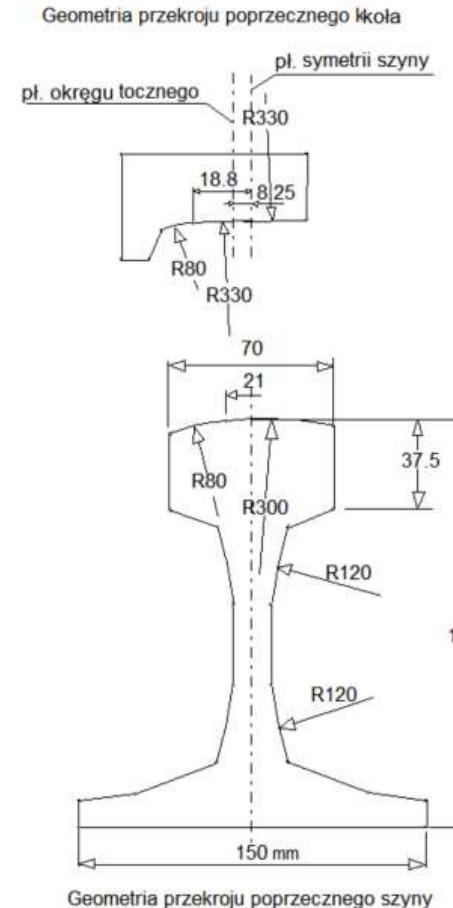
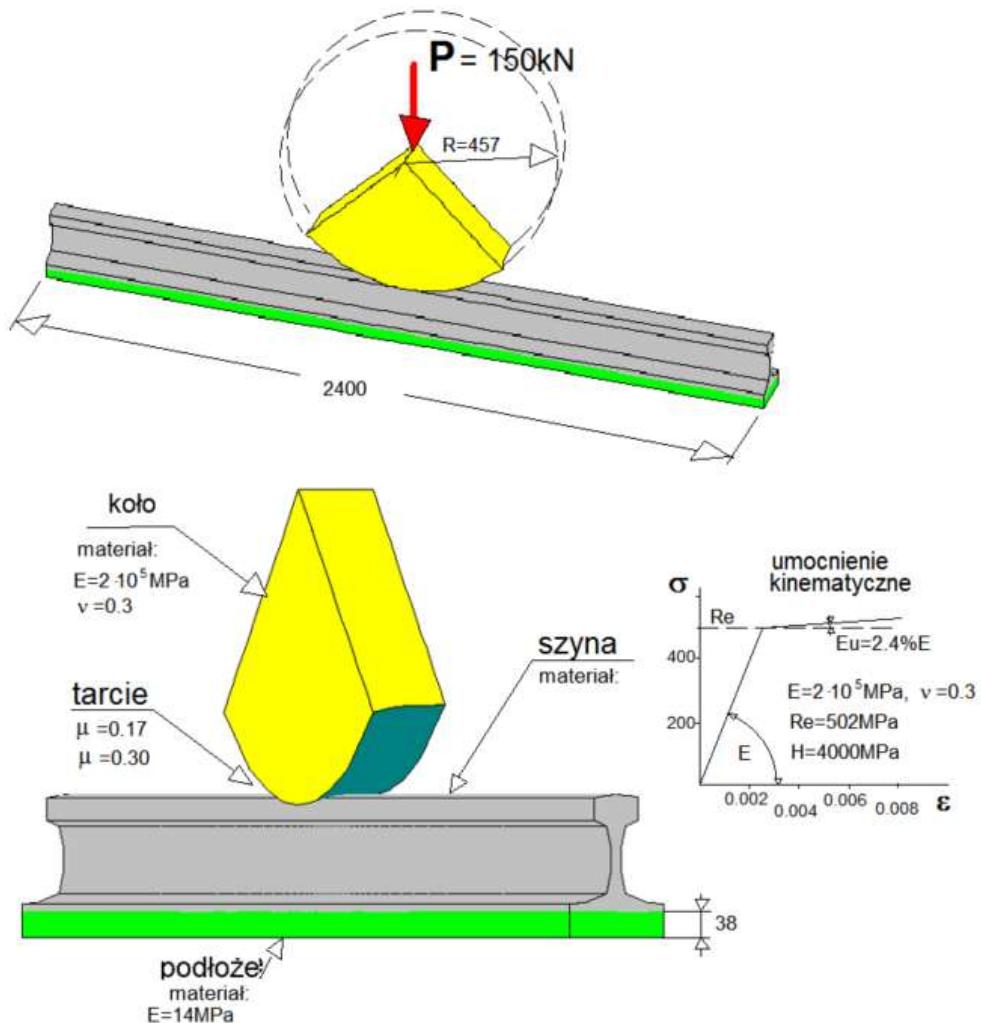


Node rotations: (a) before CBDOF command, (b) after CBDOF command

# FE ANALYSIS OF THE TURBINE BLADE LOCKING PIECE DEFECTS



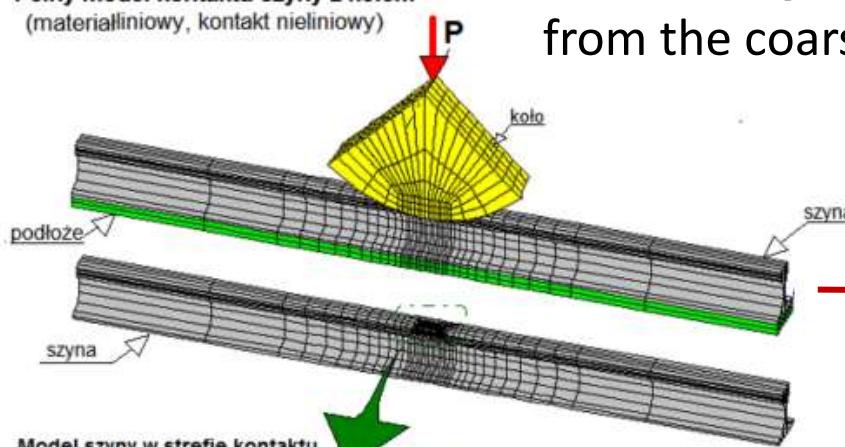
## Rough model



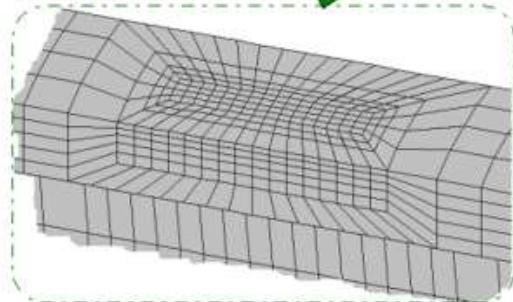
Rys.6.6. Geometria ciał i modelu materiału przyjęte w analizie kontaktu tocznego szyny z kołem.

## Transferring displacement conditions from the coarse model to the submodel

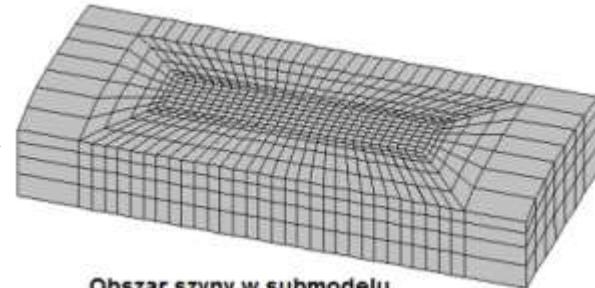
Pełny model kontaktu szyny z kołem  
(materiałliniowy, kontakt nieliniowy)



Model szyny w strefie kontaktu

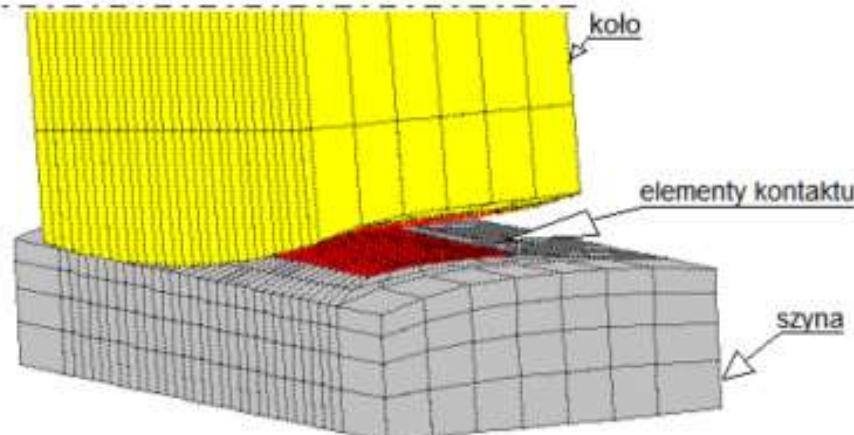


przeniesienie przemieszczeń z rozwiązania zadania pełnego (model o uproszczonej dyskretyzacji) do nowego, nieliniowego zadania kontaktu w postaci warunków na brzegach submodelu

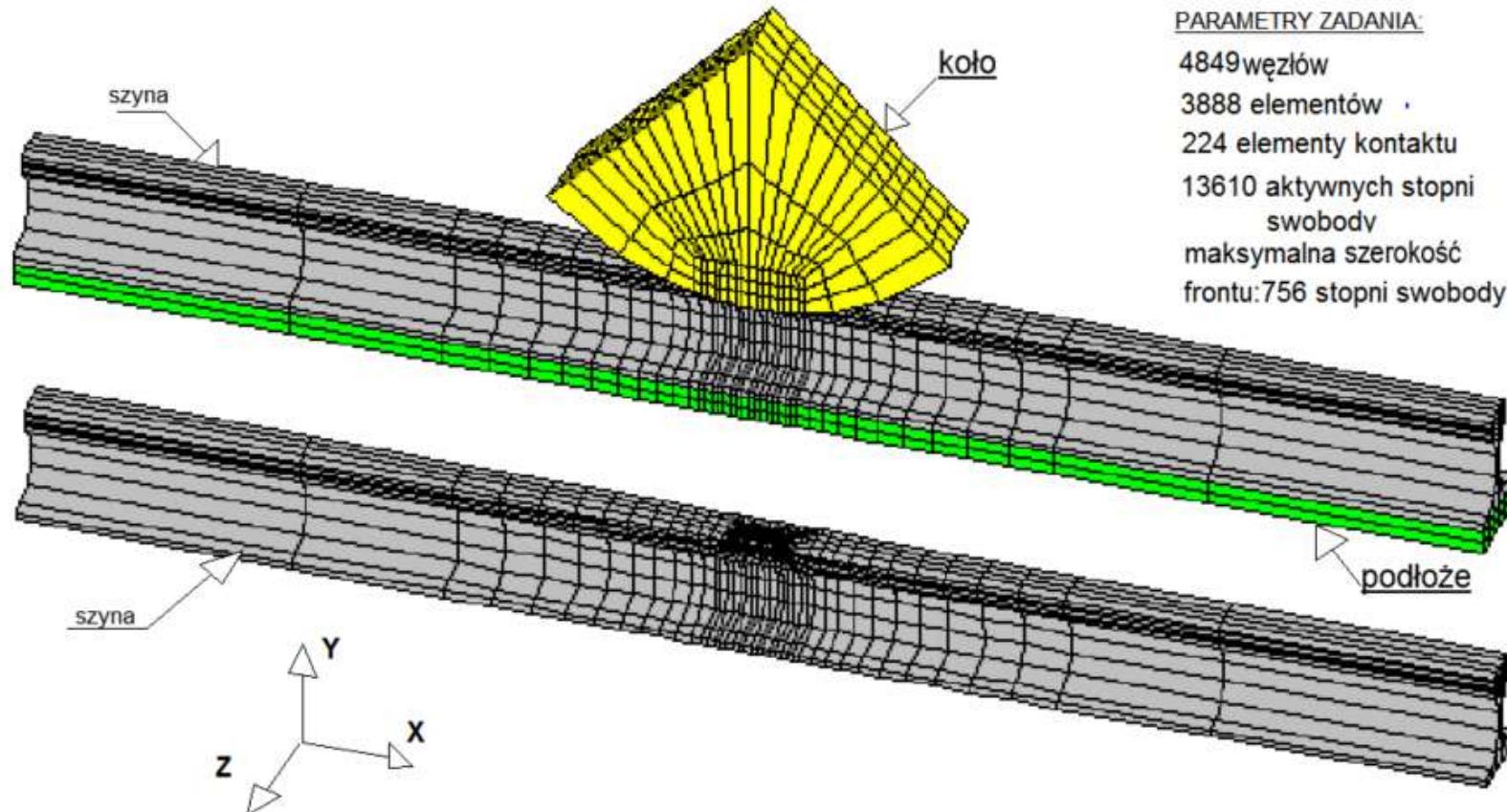


Obszar szyny w submodelu

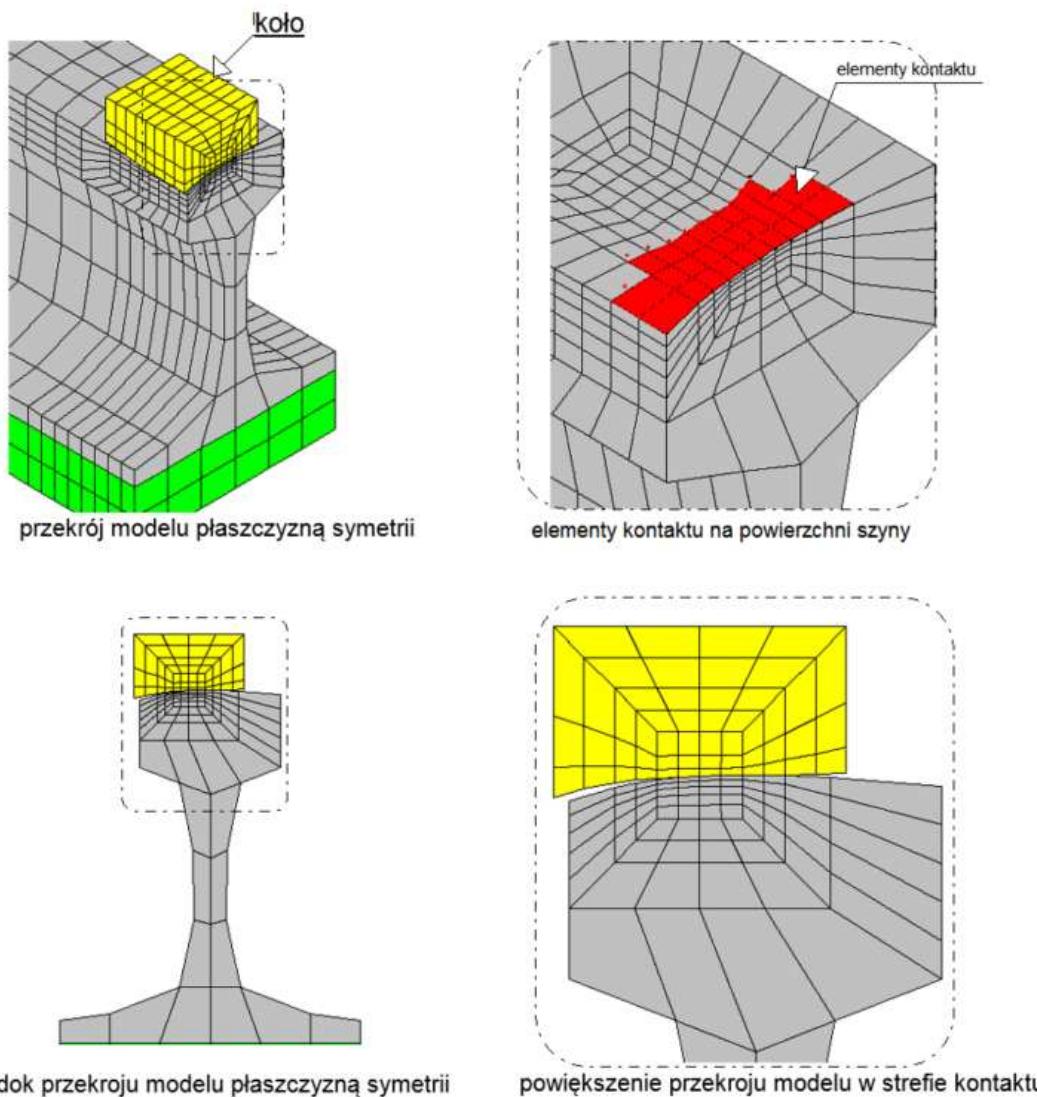
**SUBMODEL**  
**KONTAKTU SZYNY Z KOŁEM**  
(materiałszyny nieliniowy, kontakt nieliniowy z tarciem,  
obciążenie siłami i krokiem przemieszczenia)



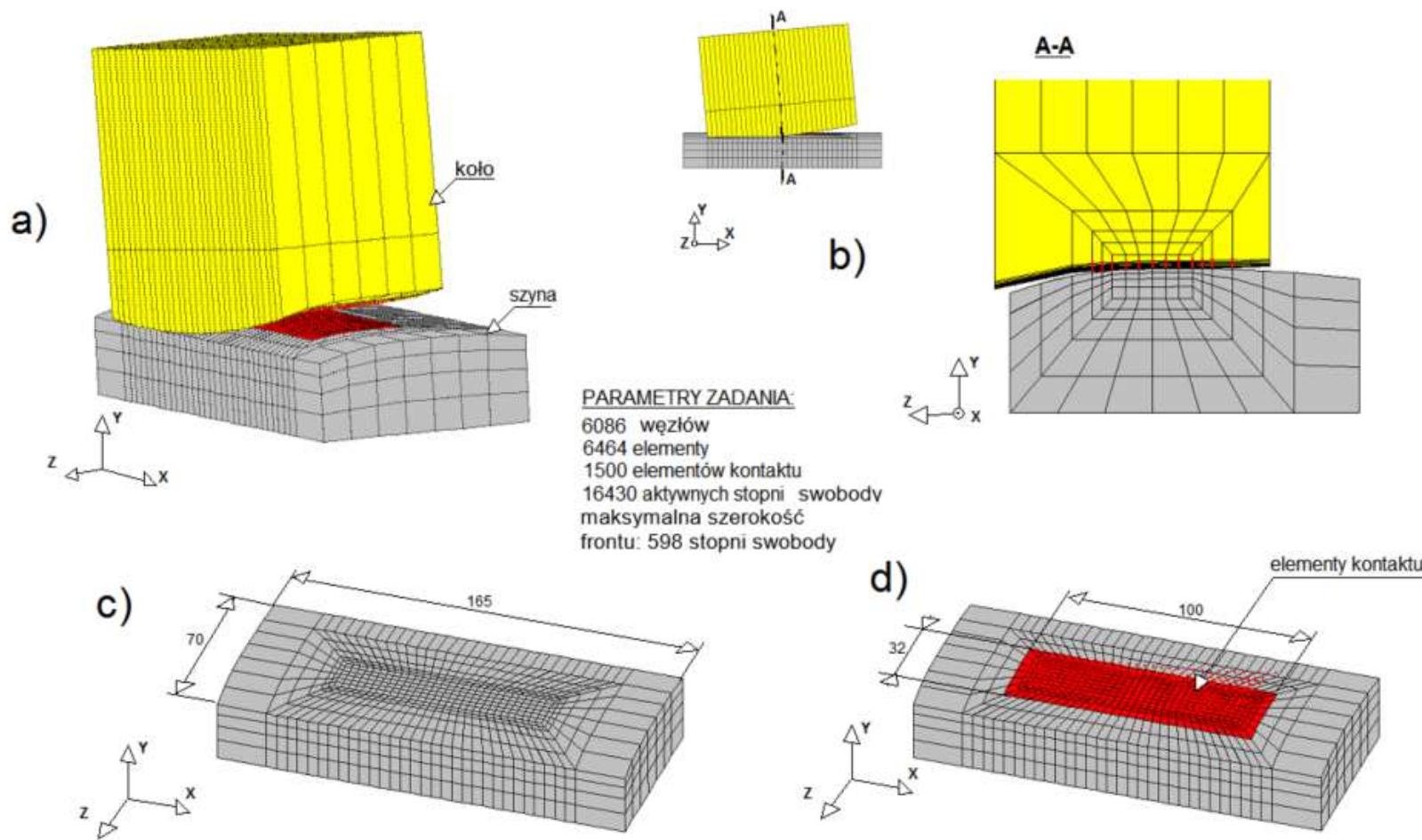
## Rough model



Budowa strefy kontaktu



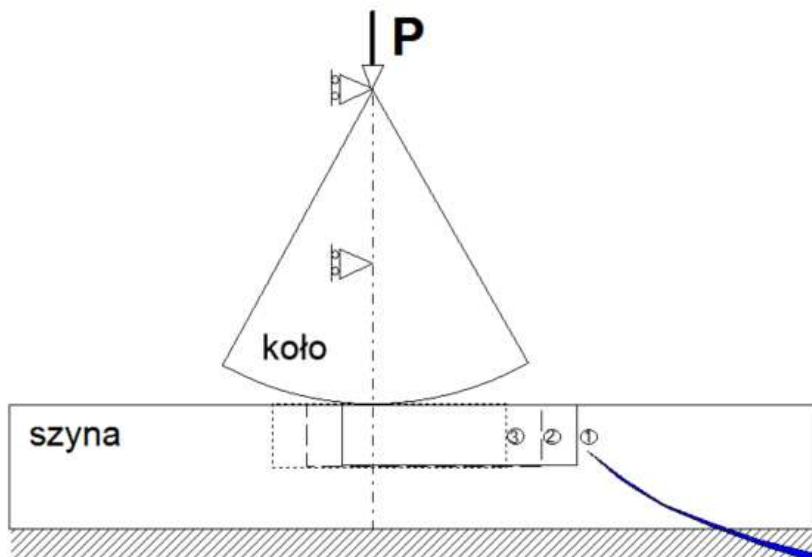
Rys.6.8. Siatka dyskretyzacyjna dla zadania pełnego kontaktu szyny z kołem.



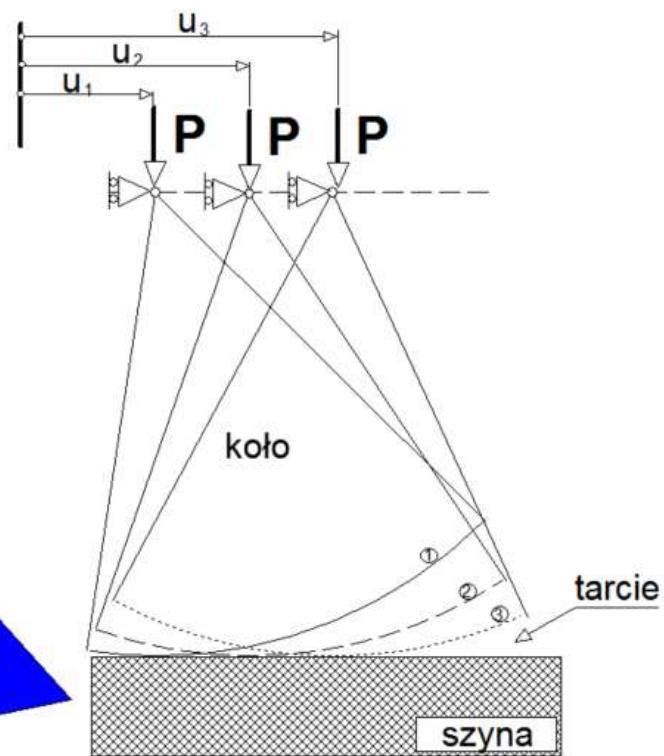
Rys.6.9. Dyskretyzacja submodelu w zadaniu kontaktu szyny z kołem;

- a) wycinek szyny objęty submodelem,
- b) przekrój submodelu płaszczyzną A-A,
- c) dyskretyzacja submodelu,
- d) dyskretyzacja strefy kontaktu z zaznaczeniem elementów kontaktu.

## MODEL

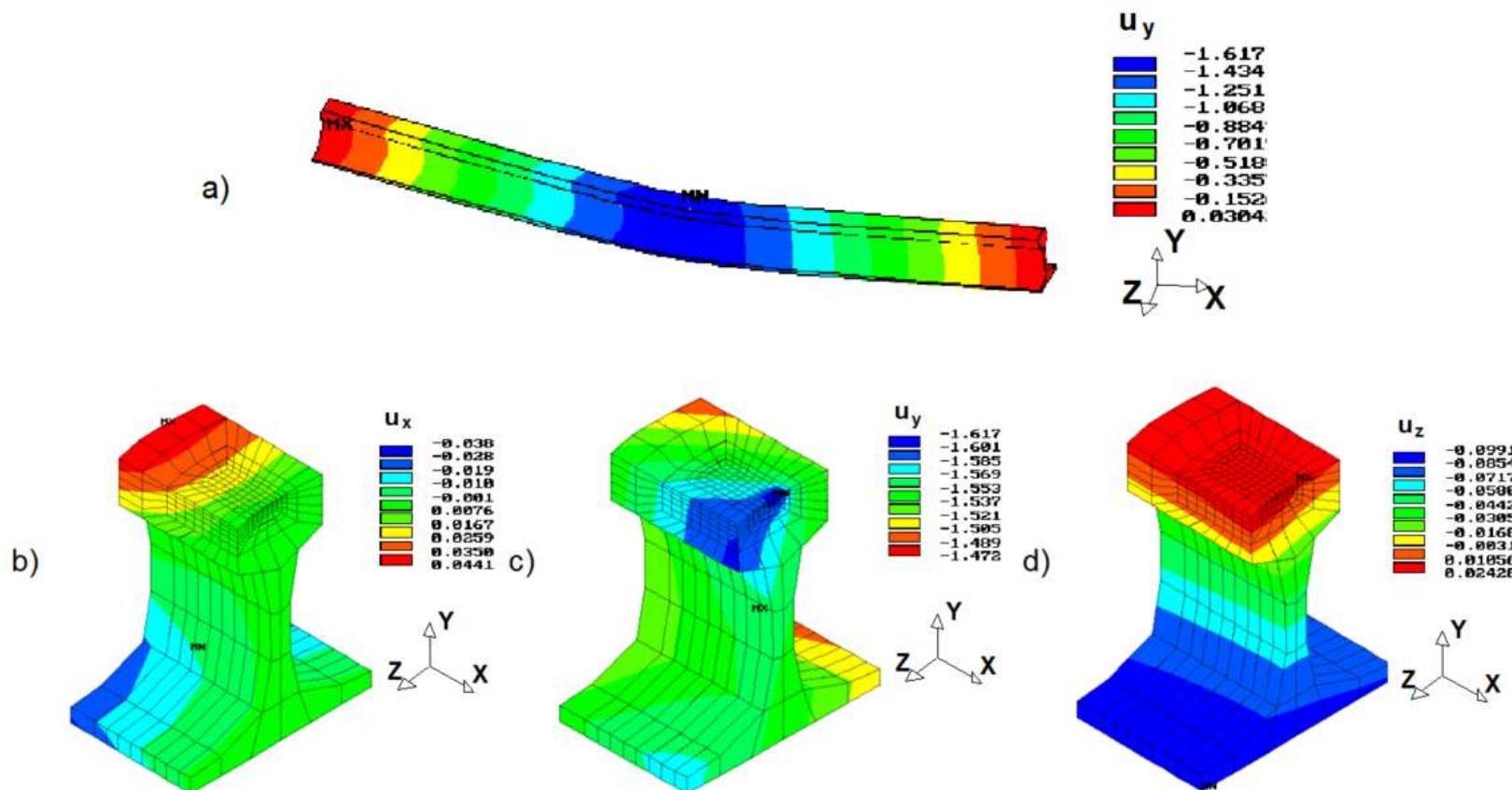


## SUBMODEL



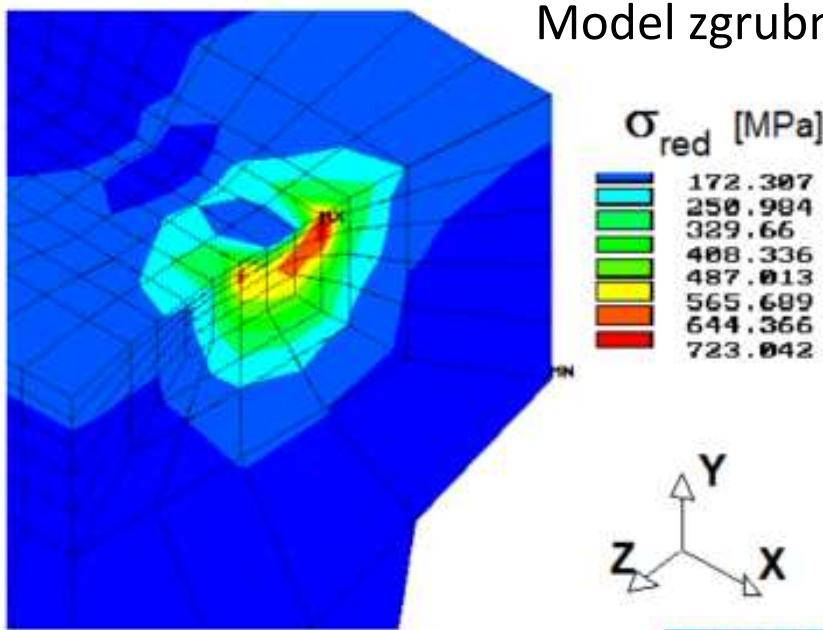
przeniesienie warunków  
brzegowych w kolejnych  
krokach obciążenia

Rys.6.10. Koncepcja toczenia w zadaniu kontaktu szyny z kołem, przy wykorzystaniu modelu i submodelu.

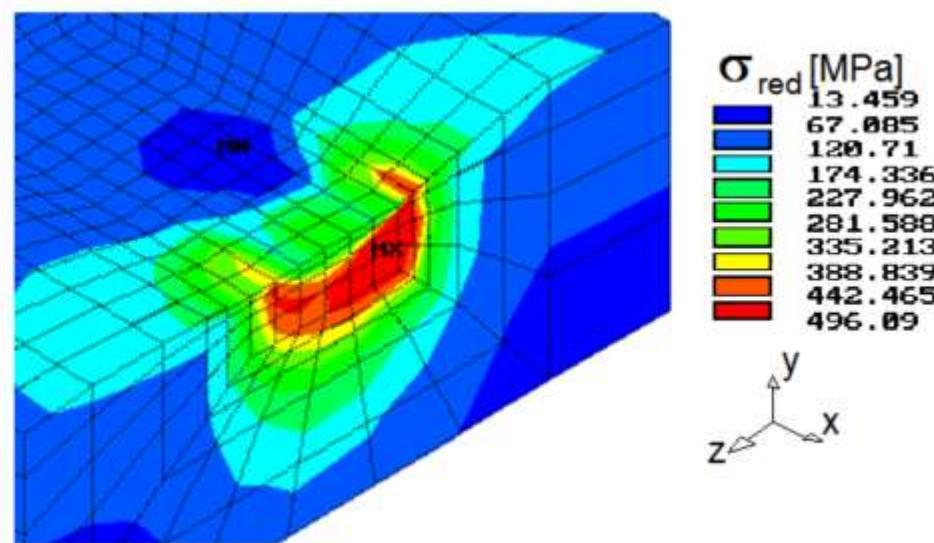


Rys.6.11. Wyniki obliczeń modelu pełnego: przebiegi składowych przemieszczenia [mm]: a) w kierunku pionowym dla całej szyny, oraz b) w kierunku wzdluznym, c) pionowym i d) poprzecznym w strefie kontaktu.

## Model zgrubny



X Y Z  
Submodel



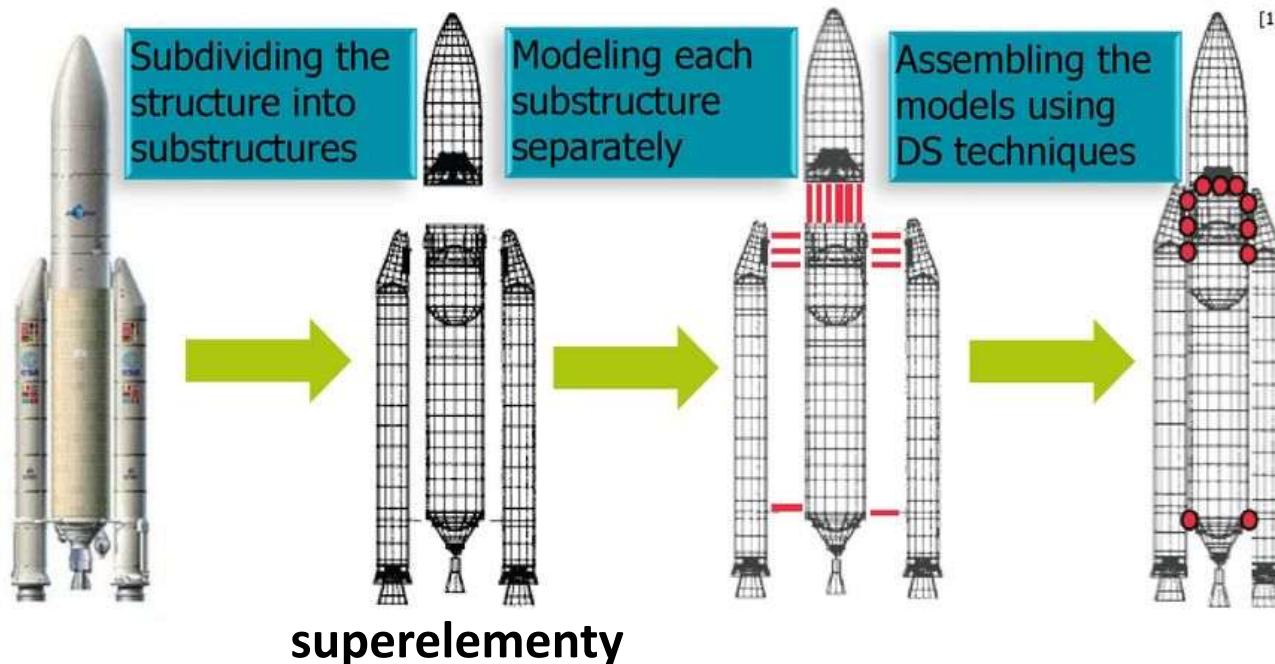
y  
X  
Z

# Substructuring

## Dynamic Substructuring

### Schematic overview

1. Introduction
2. Dynamic Substructuring (DS)
3. Experimental DS
4. Ampair 600 Wind Turbine
5. Experimental modeling
6. Assembly results
7. Conclusions
8. Recommendations



[1] Figures from Daniel J. Rixen, Dynamic Substructuring Concepts, Tutorial, IMAC 2010.

# Elements Supporting Birth and Death

The following ANSYS elements support the birth and death feature:

<a href="#">LINK1</a>	<a href="#">PLANE25</a>	<a href="#">PIPE60</a>	<a href="#">SOLID92</a>	<a href="#">TARGE170</a>	<a href="#">BEAM188</a>
<a href="#">BEAM3</a>	<a href="#">MATRIX27</a>	<a href="#">SOLID62</a>	<a href="#">SOLID95</a>	<a href="#">CONTA171</a>	<a href="#">BEAM189</a>
<a href="#">BEAM4</a>	<a href="#">LINK31</a>	<a href="#">SHELL63</a>	<a href="#">SOLID96</a>	<a href="#">CONTA172</a>	<a href="#">SOLSH190</a>
<a href="#">SOLID5</a>	<a href="#">LINK32</a>	<a href="#">SOLID65</a>	<a href="#">SOLID97</a>	<a href="#">CONTA173</a>	<a href="#">FOLLW201</a>
<a href="#">LINK8</a>	<a href="#">LINK33</a>	<a href="#">PLANE67</a>	<a href="#">SOLID98</a>	<a href="#">CONTA174</a>	<a href="#">SHELL208</a>
<a href="#">LINK10</a>	<a href="#">LINK34</a>	<a href="#">LINK68</a>	<a href="#">PLANE121</a>	<a href="#">CONTA175</a>	<a href="#">SHELL209</a>
<a href="#">LINK11</a>	<a href="#">PLANE35</a>	<a href="#">SOLID69</a>	<a href="#">SOLID122</a>	<a href="#">CONTA176</a>	<a href="#">PLANE230</a>
<a href="#">PLANE13</a>	<a href="#">SHELL41</a>	<a href="#">SOLID70</a>	<a href="#">SOLID123</a>	<a href="#">CONTA177</a>	<a href="#">SOLID231</a>
<a href="#">COMBIN14</a>	<a href="#">PLANE42</a>	<a href="#">MASS71</a>	<a href="#">SHELL131</a>	<a href="#">LINK180</a>	<a href="#">SOLID232</a>
<a href="#">PIPE16</a>	<a href="#">BEAM44</a>	<a href="#">PLANE75</a>	<a href="#">SHELL132</a>	<a href="#">SHELL181</a>	<a href="#">REINF264</a>
<a href="#">PIPE17</a>	<a href="#">SOLID45</a>	<a href="#">PLANE77</a>	<a href="#">SURF151</a>	<a href="#">PLANE182</a>	<a href="#">SOLID272</a>
<a href="#">PIPE18</a>	<a href="#">PLANE53</a>	<a href="#">PLANE78</a>	<a href="#">SURF152</a>	<a href="#">PLANE183</a>	<a href="#">SOLID273</a>
<a href="#">PIPE20</a>	<a href="#">BEAM54</a>	<a href="#">PLANE82</a>	<a href="#">SURF153</a>	<a href="#">MPC184-Link/Beam</a>	<a href="#">SOLID285</a>
<a href="#">MASS21</a>	<a href="#">PLANE55</a>	<a href="#">PLANE83</a>	<a href="#">SURF154</a>	<a href="#">SOLID185</a>	<a href="#">PIPE288</a>
<a href="#">BEAM23</a>	<a href="#">SHELL57</a>	<a href="#">SOLID87</a>	<a href="#">SHELL157</a>	<a href="#">SOLID186</a>	<a href="#">PIPE289</a>
<a href="#">BEAM24</a>	<a href="#">PIPE59</a>	<a href="#">SOLID90</a>	<a href="#">TARGE169</a>	<a href="#">SOLID187</a>	<a href="#">ELBOW290</a>